**Ch 8 - The Logic of Conditionals**

**8.11**

**If the US does not cut back on its use of oil soon, parts of California will be flooded within 50 years.**

A: US cuts back on its oil use soon.

B: US will be able to reduce its carbon emissions substantially in the next few years

C: The countries of China, India, and Brazil will join in efforts to curb carbon emissions.

D: If China, India, and Brazil don’t curb emissions then their growth is higher than if they do curb emissions.

E: China, India, and Brazil have higher rates of emissions.

F: China, India, and Brazil have higher rates of growth.

G: Higher growth rate means higher rate of emissions.

H: Greenhouse effect accelerates.

I: Sea gets warmer and ice will melt and sea level will rise.

J: Low lying coastal areas will be subject to flooding within 50 years.

K: Parts of California will be flooded.

**Implicit premises**

~A -> ~B, or B -> A

If the US is able to reduce carbon emissions then the US will cut back on its oil use soon.

The US is able to reduce carbon emissions only if it will cut back on its oil use soon.

~B -> ~C or C -> B

China, India and Brazil will join efforts to cut emissions only if the US is able to reduce its emissions substantially in the next few years.

~C -> F

If China, India, and Brazil don’t join efforts to curb emissions, then they will have higher rates of growth.

F -> E

If China, India, and Brazil have higher rates of growth, then they have higher rates of emissions.

E -> H

If they have higher rates of emissions then the greenhouse gas accelerates.

H -> I

If the greenhouse gas accelerates, then the sea gets warmer and sea levels rise.

I -> J

If sea levels rise then low lying coastal areas get flooded.

J -> K

If low lying coastal areas get flooded then parts of Cali will get flooded.

Assume ~A.

Then ~B.

Then ~C.

Then F.

Then E.

Then H.

Then I.

Then J.

Then K.

**The implicit premises are all plausible.**

**8.15**

Assume n is divisible by 3.

Then we can write n = 3m, for some natural number m.

Then n^2 = 9m^2, which is divisible by 3, and also by 9.

By conditional introduction, n is divisible by 3 -> (n^2 is divisible by 3) ^ (n^2 is divisible by 9).

Assume n is divisible by 3.

Then n=3m.

n^3 = 27m^3, divisible by 3, 9, and 27.

By conditional introduction, n divisible by 3 -> n divisible by 3,9,27

**8.39**

* if we can show that a conclusion is a tautological consequence of certain premises (which we can do using a truth table), then we know that there is a proof of the argument (as per the Completeness Theorem)

**8.41**

* Assume there is a proof of S given A1, …, An.
* Assume that in that proof, there is a step that is not a tautological consequence of the premises and assumptions in force at that point in the proof.
* **Case ~ Elim**: Assume the invalid step is the result of application of negation elimination. Therefore, we have a sentence Q that results from application of this rule to a sentence ~~Q.
* Make a truth table of A1, …, An, ~~Q, and Q
* Because Q is not a tautological consequence of A1, …, An and ~~Q, there must be a row wherein A1, …, An, and ~~Q are all true but Q is false. But if ~~Q is true, then Q is true true. So we have a contradiction.

**8.42**

* **Case ~Intro:** Assume the invalid step is the result of application of ~ Elim. Therefore, there is a subproof with assumption P that results in a contradiction, and there is a step ~P that results from application of ~Elim to the subproof, but ~P is by assumption not a tautological consequence of the premises in force at that point, namely A1,..., An.
* Construct a truth table of A1,..., An, P, and ~P.
* There must be a row wherein A1,...,An are all true and ~P is false. But then P is true. But if A1,...,An, and P are all true, then the subproof showed that a contradiction ensues.
* Therefore, there is a contradiction: it is not possible for a step to be inferred using negation introduction but to not be a tautological consequence of the premises.

**8.43**

* **Case | Elim:** Assume the invalid step is the result of application of | elimination, ie disjunction elimination, ie the result of a proof by cases.
* Then there are n subproofs that all result in some sentence Q, and the invalid step asserts Q based on the subproofs,a disjunction of n case premises P1,..., Pn, and premises A1,...,An.
* Construct at truth table for A1,...,An, P1 | … | Pn, Q
* There must be a row wherein A1, …, An are all true, at least one of P1, …, Pn is true (such that the disjunction is true), and Q is false.
* Say Pm is true among the P1, …, Pn. Then A1, …, An, Pm are true, so Q is true.
* But this contradicts our statement that Q is false, that results from our assumption that Q is an invalid step after application of | elimination.

In any of the cases in the Soundness Theorem proof, we end up with a contradiction. By disjunction elimination we conclude that a contradiction ensues at the level of the subproof that has as assumption that there is a step that is not a tautological consequence of the premises and assumptions in force at that point in the proof.

By negation introduction, we conclude that assuming there is a proof of S given A1, …, An, S is always a tautological consequence of the premises A1, …, An, and assumptions in force at the point in the proof in which S appears.